Discrete choice modeling without regret

Caspar Chorus

Delft University of Technology

A bit about myself

Chair:
Choice behaviour modelling

Head:
Transport and Logistics Group (Fac. Of Techn., Policy and Manag.)

Background, positioning:
Somewhere in between Econometrics and Behavioural sciences

Research (and teaching) aim:
- Improve behavioural realism of Discrete choice models
- While maintaining high levels of Econom(etr)ic tractability

Area(s) of application:
Travel behaviour + marketing / health / **environment** / politics / …

A bit about linear-additive RUM-models

Notion of linear-additive utility maximization may be unrealistic

- Acknowledged by most choice modelers, and even some neoclassical Economists/Econometricians

- How about loss aversion, reference-dependency?
  - E.g. Prospect Theory suggests that reference points matter, and that loss weigh heavier than gains

- How about choice set effects?
  - Composition of the choice set is known to influence choice behaviour in subtle ways (decoy effects such as the compromise effect)

The **Random Regret Minimization (RRM)** model was designed to capture both behavioral phenomena, while remaining econometrically tractable and parsimonious

Part I

Random Regret Minimization

Chorus, C., van Cranenburgh, S., & Dekker, T. (2014). Random regret minimization for consumer choice modeling: Assessment of empirical evidence. *Journal of Business Research*, *67*(11), 2428-2436.

Thiene, M., Boeri, M., & Chorus, C. G. (2012). Random regret minimization: exploration of a new choice model for environmental and resource economics. *Environmental and resource economics*, *51*(3), 413-429.

Random Regret Minimization: Contrasts with previous regret models…

Regret Theory, MiniMax regret: Popular in microeconomics

- Focuses on single-attribute decision-making ('money')
- Focuses on binary choice sets ('lottery')
- Developed to capture anomalies in risky choice
- Axiomatic foundation in Decision Theory

Random Regret Minimization: Designed for discrete choice-modelers

- Focuses on multi-attribute decision-making
- Focuses on multinomial choice sets
- Developed to capture loss aversion, reference dependency, choice set composition effects in riskless choice
- Pragmatic foundation in Discrete Choice-econometrics

Random Regret Minimization: core assumptions

- People choose the alternative with minimum regret
- Regret associated with a considered alternative equals sum of regrets associated with binary comparisons with all other alternatives
- Binary regret equals sum of regrets associated with comparing the considered alternative with another alternative, on each of their attributes
- Attribute-regret: **convex** function of attribute difference
  - Avoiding weak performance (relative to competition) more important than
  - Attaining strong performance (relative to competition)

RRM – mathematical notation

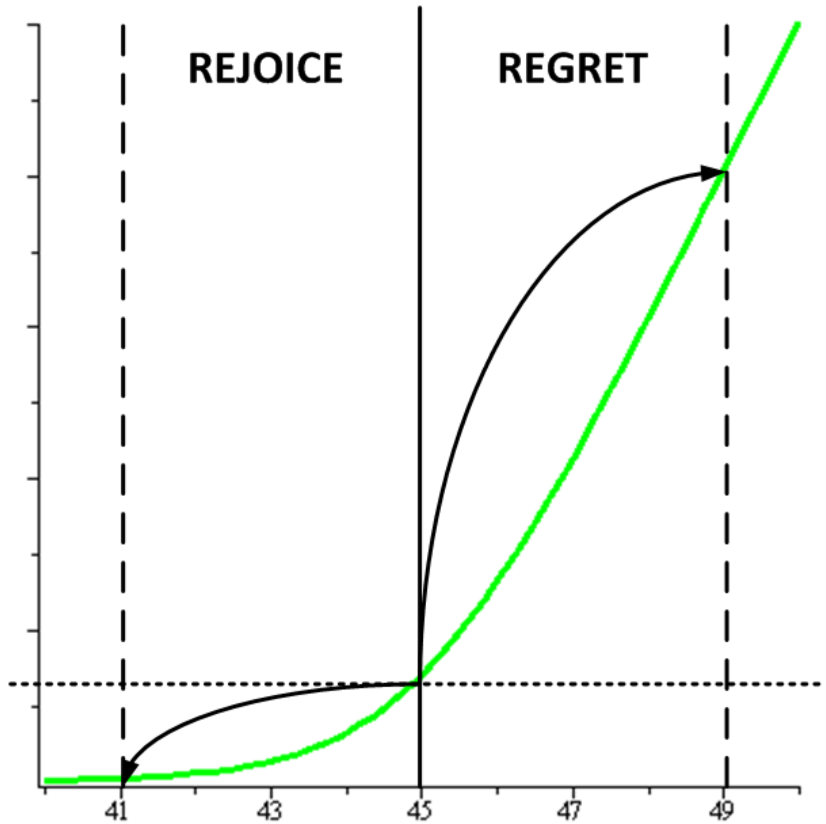
$$R_i = \sum_{j \neq i} \sum_m \ln\left(1 + \exp\left[\beta_m \cdot \left(x_{jm} - x_{im}\right)\right]\right)$$

- i: Regret of Alternative i
- $j \neq i$: Repeat and sum over all <u>competing</u> alternatives
- m: Repeat and sum over all attributes
- β_m: Weigh, according to importance of attribute m (estimable parameter! can be <0)
- $x_{jm} - x_{im}$: Compare i's performance on attribute m, with j's

Chorus, C. G. (2010). A new model of random regret minimization. *European Journal of Transport and Infrastructure Research*, 10(2), 181-196.

Attribute-regret:
Convex function of attribute-difference

- Route A is compared to route B
- In terms of travel time
- B's travel time = 45 mins
- A's travel time is varied
- A's regret is plotted



Observations:

- Travel time increase matters more than decrease
  avoiding regret is more important then achieving rejoice
- Relative position wrt reference point (45 mins) matters
  when initial (relative) performance is worse, effect of deterioration is bigger.

RRM: Logit-choice probabilities

$$R_i = \sum_{j \neq i} \sum_m \ln\left(1 + \exp\left[\beta_m \cdot \left(x_{jm} - x_{im}\right)\right]\right)$$

Binary attribute-regret, summed over attributes, competing alternatives.

$$RR_i = R_i + v_i = \sum_{j \neq i} \sum_m \ln\left(1 + \exp\left[\beta_m \cdot \left(x_{jm} - x_{im}\right)\right]\right) + v_i$$

If $-v_i$ distributed i.i.d. Extreme Value Type I, then Logit-probabilities:

$$P(i) = P\left(RR_i < RR_j, \forall j \neq i\right) = P\left(-(R_i + v_i) > -(R_j + v_j), \forall j \neq i\right)$$

$$= \frac{\exp(-R_i)}{\sum_{j=1..J} \exp(-R_j)}$$

RRM-model: Properties

Reference-dependent convexity of regret function implies:

1. Performance of other, ‘irrelevant’ alternatives matters

2. Performing well weighs less than performing poorly

Combined, these two core properties of RRM give rise to its trademark property: the RRM model captures compromise effects (preference for alternatives that are positioned ‘in the middle of the choice set’)

Chorus, C.G., Bierlaire, M., 2013. An empirical comparison of travel choice models that capture preferences for compromise alternatives. *Transportation*, 40(3), 549-562

RRM: an **addition** to the toolbox

- Incorporated in NLOGIT, LatentGOLD, Sawtooth, (Biogeme), …
- Widely covered in textbooks (Hensher et al., 2015), courses (UK, US, Aus)
- And used in dozens of empirical applications.

- All sorts of mobility choices (mode, route, departure time, parking lots, etc.)
- Evasive actions on highways (preceding accidents)
- Vehicle purchases (regular cars, alternative fuel vehicles)
- Travel information service usage
- Freight movement (travel mode)
- Policy choices by politicians, voting behavior
- Shopping destinations
- Workplace locations
- Nature park visits / tourism destination choices
- Choices for medical treatments of patients
- Lifestyle / dietary choices
- Poaching behavior (Tanzania)
- On-line dating behavior
- …

Summary of empirical performance:

RRM vs. RUM: 50/50.

Differences often significant, but modest.

Although choice probability differences usually larger for particular choice situations

Part II

Random Regret Minimization: New insights (μRRM)

van Cranenburgh, S., Guevara, C.A., Chorus, C.G., 2015.
New insights on random regret minimization models.
Transportation Research Part A, 74, 91-109

The µRRM model – formal

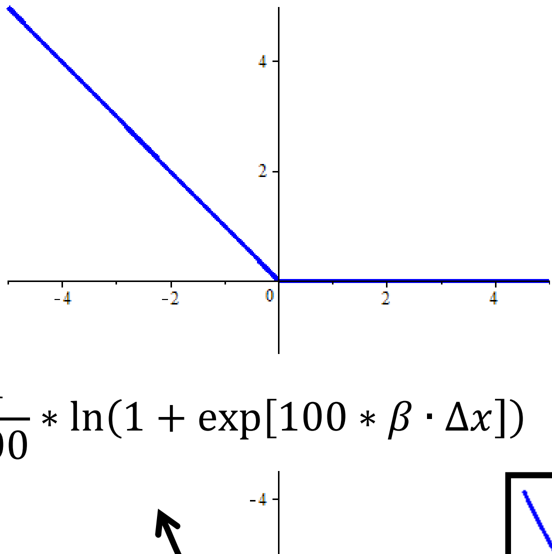
$$R_i = \sum_{j \neq i} \sum_m \boldsymbol{\mu_m} \cdot \ln\left(1 + \exp\left(\frac{\beta_m}{\boldsymbol{\mu_m}}\left[x_{jm} - x_{im}\right]\right)\right)$$

Special cases:

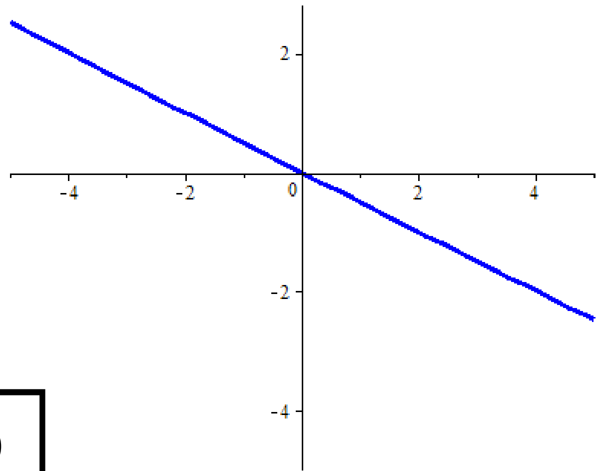
- $\mu \to 0$: only regret matters; rejoice is irrelevant. 'Pure-RRM'.
- $\mu = 1$: conventional RRM (Chorus, 2010)
- $\mu \to +\infty$: regret and rejoice matter equally; linear-additive RUM. $\hat{\beta}_m^{RUM} \cong \frac{1}{2} J \hat{\beta}_m^{\mu RRM}$ (where J is choice set size)

The μRRM model – visual ($\beta = -1$)

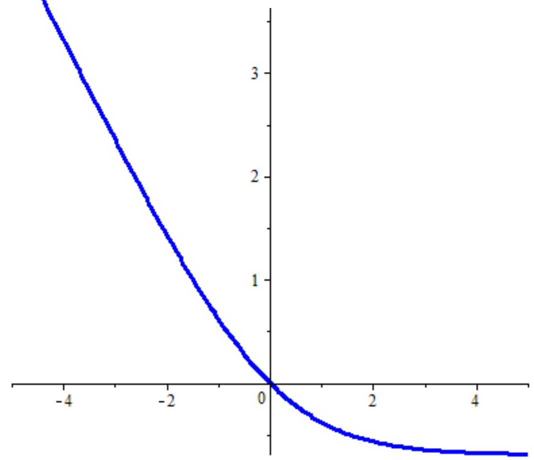
$$\frac{1}{100} * \ln(1 + \exp[100 * \beta \cdot \Delta x])$$



$$100 * \ln\left(1 + \exp\left[\frac{1}{100} * \beta \cdot \Delta x\right]\right)$$



$$\ln(1 + \exp[\beta \cdot \Delta x])$$



Note: constant added, to ensure regret goes through origin.

Check vertical axes: **Attribute importance stays (roughly) the same.**

The μRRM model – interpretation

μ is an estimable parameter of regret aversion

jointly with β, which measures attribute importance

Whereas these were lumped together in the restrictive 2010-version of RRM

Result: flexible regret function

which identifies attribute importance and regret aversion

$\rightarrow$ Capable of capturing extreme regret aversion (i.e., irrelevance of rejoice)

$\rightarrow$ Nests the linear in parameters RUM model

The µRRM model – Mathematical

- $\ln(1+\exp[\beta \cdot \Delta x])$ originally proposed as a smoothing-function of $\max\{0, \beta \cdot \Delta x\}$
- max-operator caused difficulties with model estimation, derivation of WtP, etc.
- two iid EV Type I-errors added to 0 and $\beta \cdot \Delta x$, respectively; integrated out.
- results in Logsum-form (ignoring cnst.): $E\left[\max\left(0+\nu_1, \beta \cdot \Delta x+\nu_2\right)\right]=\ln\left(1+\exp[\beta \cdot \Delta x]\right)$
- in doing so, it was implicitly assumed that error-variances (ν) normalized to $\pi^2/6$.
- this implicit assumption can be relaxed: variance of implicit errors can be estimated.
- if variance of $\nu = (\pi^2/6) \cdot \mu^2$, $E\left[\max\left(0+\nu_1, \beta \cdot \Delta x+\nu_2\right)\right]=\mu \cdot \ln\left(1+\exp\left[\frac{\beta}{\mu} \cdot \Delta x\right]\right)$
- small (large) variance of implicit errors implies kink (smooth transition) around zero.
- as such, μ determines the ‘smoothness’, or linearity, of the regret function.

Estimating µRRM – shopping location

Model	RUM		Classical RRM		P-RRM		µRRM	
Final Log-likelihoog	-2305.2		-2300.9		-2278.5		-2262.6	
Number of parameters	3		3		3		4	
ρ^2	0.047		0.049		0.058		0.065	
Parameters	*Est*	*t-stat*	*Est*	*t-stat*	*Est*	*t-stat*	*Est*	*t-stat*
Floor_space_Groceries	0.106	6.690	0.068	6.766	0.146	11.92	0.131	11.615
Floor_space_Other	0.011	4.978	0.003	2.777	-0.001	-0.302	0.001	1.1825
Travel_Time	-0.045	-8.961	-0.016	-8.337	-0.010	-5.886	-0.012	-6.926
μ							0.139	87.83 [a]

[a] *t*-test for difference from one

Estimating µRRM – shopping location (II)

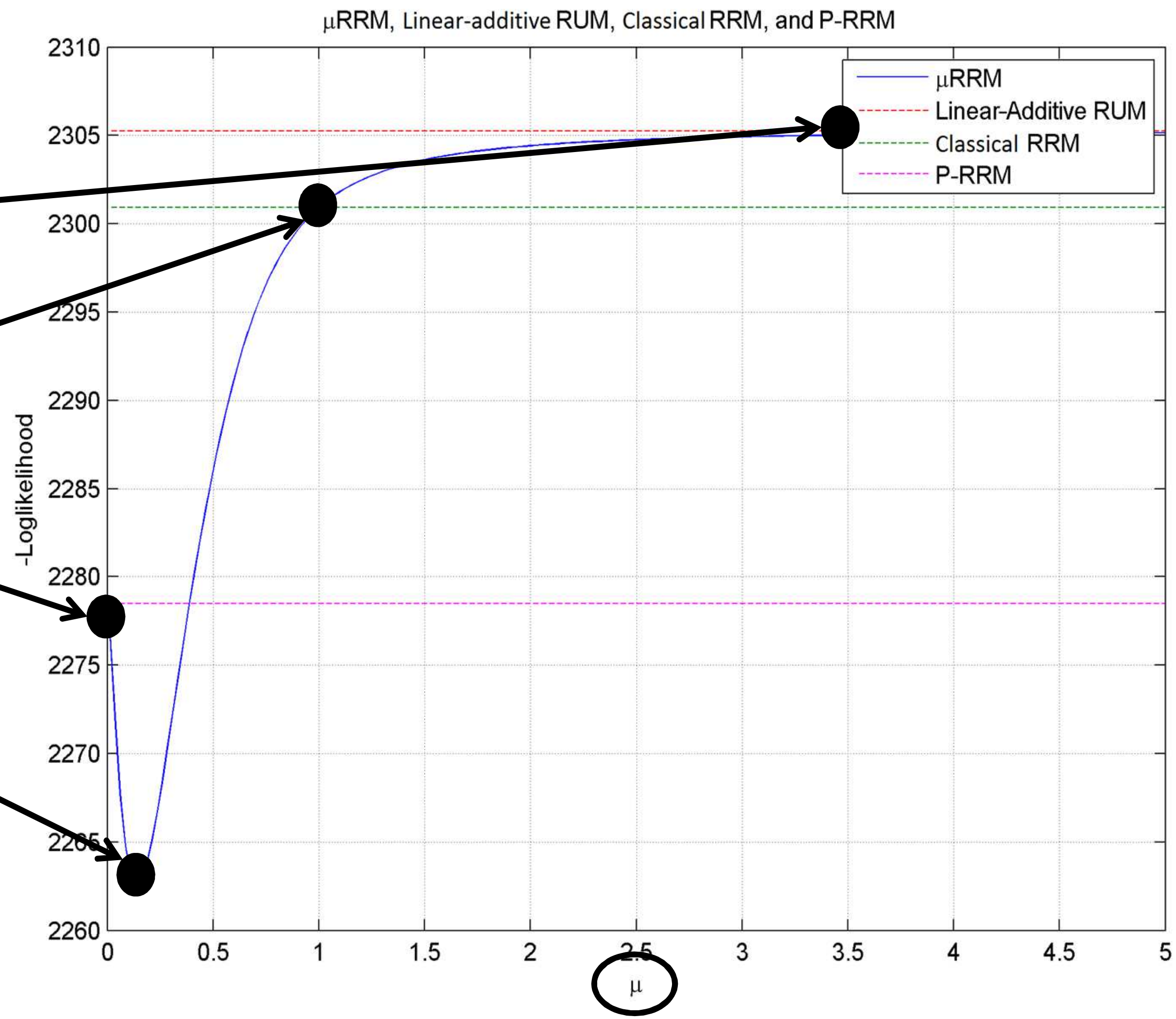
Estimation for diff. values of μ:

Linear RUM fits worst.

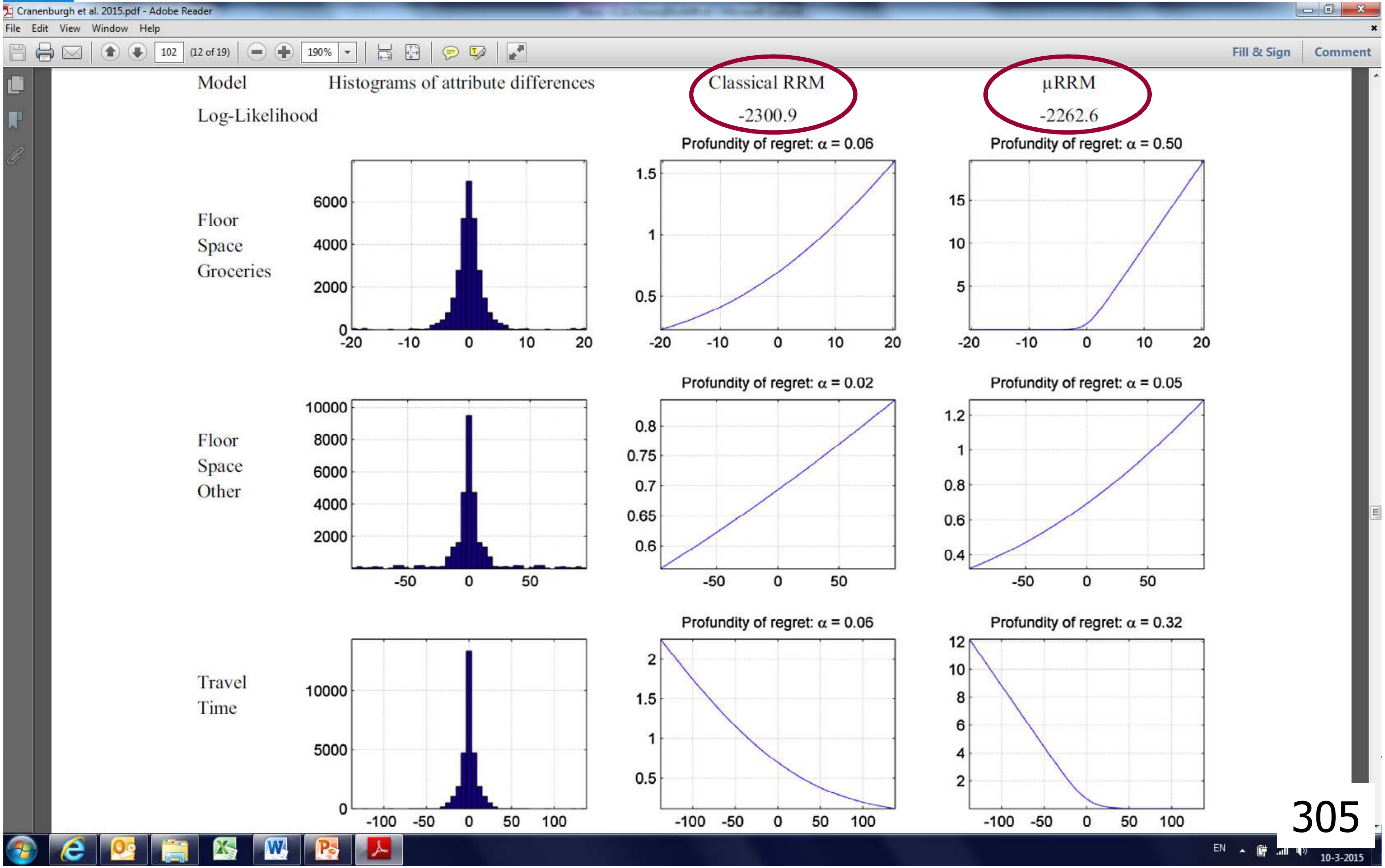
Conventional RRM does somewhat better.

Pure-RRM does a lot better.

But the best fit is for a model that approaches, yet not equals, Pure-RRM.



Estimating μRRM – shopping location (III)



Estimating μRRM – 10 datasets

Revisited 10 datasets used in previous publications to compare RRM, RUM.

- On 6 out of 10 datasets, conventional RRM outperformed RUM.
- On 4 out of 10, RUM fitted the data better.
- Differences usually significant, but with one exception, small or modest.

Results based on μRRM :

- For datasets where RUM did better than conventional RRM, μRRM reduces to RUM.
- Of the 6 datasets where conventional RRM did better than RRM:
  - On 2 datasets, μRRM reduces to conventional RRM
  - On 3 datasets, μRRM achieves values in-between conventional RRM and Pure-RRM
  - On 1 dataset, μRRM reduces to Pure-RRM
- For the last 4 datasets, model fit improvement found to be very substantial
  - At the cost of one extra parameter
  - Out of sample performance in line with GoF

μRRM – Conclusions

- Alleviates a restrictive assumption underlying RRM's functional form
- Nests linear RUM, conventional RRM, Pure-RRM
- Added flexibility dis-entangles regret aversion from attribute importance
- Added flexibility potentially results in large increases in model fit
- Data, code (Matlab, Biogeme), examples available at http://www.advancedrrmmodels.com/ (Sander van Cranenburgh)

Work to be done:

- Comparing μRRM with RUM, non-linear models, on different datasets
- Allow μ to differ between attributes
- Parameterize μ, to explore determinants of regret-minimization behavior
- Incorporate in Latent Class approach (allowing μ to vary across classes)

Part III

Random Regret Minimization: Issues wrt economic appraisal

(Based on joint work with Thijs Dekker, paper currently being revised for publication)

Consumer Surplus for linear RUM

Suppose with some policy you change the utility of alternative i by some very small amount ∂V_i.

The impact on welfare then equals ∂V_i if i is chosen, and 0 otherwise.

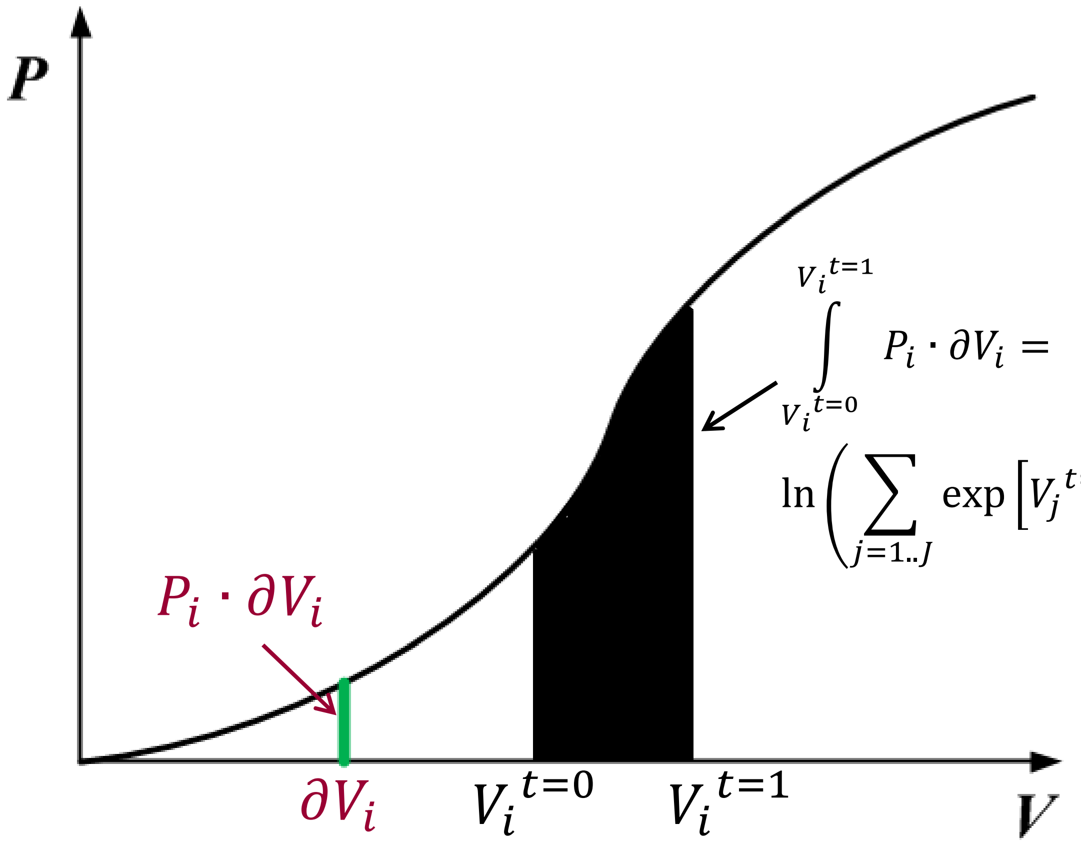
So, welfare gain associated with ∂V_i is measured by $P_i \cdot \partial V_i$.

Then, impact on welfare of larger change from ${V_i}^{t=0}$ to ${V_i}^{t=1}$ is given by the integral of the choice probability function between ${V_i}^{t=0}$ and ${V_i}^{t=1}$

(that is: every marginal change ∂V_i is weighted with the probability P_i that a randomly sampled individual experiences the change)

In other words, difference in welfare equals difference in 'area underneath probabilistic demand curve'; for Logit model, this results in a Logsum-difference.

Consumer Surplus for linear RUM (II)



$$\int_{V_i^{t=0}}^{V_i^{t=1}} P_i \cdot \partial V_i = \ln\left(\sum_{j=1..J} \exp\left[V_j^{t=1}\right]\right) - \ln\left(\sum_{j=1..J} \exp\left[V_j^{t=0}\right]\right)$$

Consumer Surplus for linear RUM (III)

Associated gain in Welfare (i.e., in Expected Utility) equals:

$$\ln\left(\sum_{j=1..J} \exp\left[{V_j}^{t=1}\right]\right) - \ln\left(\sum_{j=1..J} \exp\left[{V_j}^{t=0}\right]\right)$$

But: welfare gain or benefits associated with the policy now measured in utilities, while costs are in € → no trade-off possible. Solution: divide by marginal utility of income (γ: util / €) to give diff. in Consumer Surplus.

$$\Delta CS = \frac{1}{\gamma}\left[\ln\left(\sum_{j=1..J} \exp\left[{V_j}^{t=1}\right]\right) - \ln\left(\sum_{j=1..J} \exp\left[{V_j}^{t=0}\right]\right)\right]$$

Issue: γ not estimable. Neg. of travel cost parameter may be used instead.

(Issue: assumes no income effects. OK for relatively small policy effects.)

RRM: Problems with appraisal

Two issues which so far have hampered derivation of consistent Logsum-based Consumer Surplus measures for RRM:

1. No such thing as 'marginal regret of income'

- Adding x euros to price of all alternatives leaves regret levels unchanged (since regret is a function of price-**differences**)
- So, no way to translate regret differences into monetary terms

2. Changes in an alternative's attributes affect all alts.' regrets

- So, impact of A's travel time increase influences B's regret;
- This implies that changes in regrets of **all alternatives** have to be considered, when computing change in choice set regret…

A solution for 'issue 1'

'Forgotten' insight from Environmental Econ. (McConnel, 1995):

- Derive CS directly in monetary terms
- Circumvent in-between step (utility terms)

Approach explained for the case of an alternative's existence value

(how valuable is the mere presence of the alternative?)

1. Levy a hypothetical tax on top of the alternative's price
2. Integrate probabilistic demand over the tax, until $+\infty$
3. Interpretation: 'tax prices the alternative out of the market'
4. Gives monetary existence value of alternative: $\int_0^\infty P(\text{tax})d\text{tax}$

McConnel, 1995: equivalent to Logsum-approach for linear RUM.
Works for RRM as it relies on prices, not utility/income.

A solution for 'issue 1' (II)

McConnell (1995) approach predicts meaningful differences in existence value between RUM, RRM.

Tabel: WTP for compromise alternatives

	Route A	Route B	Route C
P-RUM	70%	23%	7%
P-RRM	67%	27%	6%
CS-RUM	€6,97	€1,48	€0,44
CS-RRM	€5,49	€1,62	€0,38

1	Route A	Route B	Route C
Average travel time	45	60	75
Percentage of travel time in congestion	10%	25%	40%
Travel time variability	±5	±15	±25
Travel costs	€12,5	€9	€5,5
YOUR CHOICE	□	□	□

Note: route B is a compromise alternative, as it has an intermediate performance on every attribute; A and C are 'extreme' alternatives.

A (very) partial solution for 'issue 2'

Changes in an alternative's attribute(s) affect all alternatives' regrets

- No problem for derivation of (changes in) value of an alternative; like in case of existence value.
- Problematic for derivation of (changes in) value of a choice set; and this is what policy makers care about most.

RRM: **not** sufficient to know $P_i \cdot \partial R_i$, along the 'policy-path' (e.g. price change), since **all regrets** change following i's price change.

- **Change in one alt.'s attribute:** Difference in existence value of the alternative before and after the change gives upper bound (improvement), respectively lower bound (deterioration) of difference in CS at the choice set level.
- **Change in multiple alternatives, attributes:** path-dependency precludes derivation of CS at the choice set level.

RRM for economic appraisal: Conclusions

RRM: not so fertile ground for economic appraisal.

No ‘marginal regret of income’, subtle impacts at choice set level.

- Some progress (is being) made: Existence value, but also RRM-VoT (Dekker, 2014)
- But **much** work still to be done – you are cordially invited!

My personal view:

- RRM is a model of <u>behavior</u>, not (or: less) a model for <u>valuation</u>.
- Linear RUM is perfect for valuation, but less realistic as a model of behavior.
- RRM’s upside (reference-dependency, choice set effects) is also its downside.
- All of this holds for many other non-RUM models (RAM, CCM, etc.) as well.
- And: note that RUM-economic appraisal also becomes very difficult when marginal utility of income is assumed to be non-linear.